

Math 1, Lecture 3
Precalculus

Midterm 1

Instructions: You have 50 minutes to complete the exam. There are five problems, worth a total of fifty points. You may not use any books, notes, or calculators. Partial credit will be given for progress toward correct solutions.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: Solutions

Section: _____

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Problem 1.

Let $C(t)$ a function which describes the cost of your cell phone plan based on the number of minutes you talk. Answer each of the following questions in a sentence.

- (a) [3pts.] What does $C(82) = 15$ mean?
- (b) [3pts.] What does $C^{-1}(20)$ represent?
- (c) [4pts.] What would it mean to say that a reasonable domain for $C(t)$ is $[0, 500]$?

(a) If you talk 82 minutes, the cost is \$15.

(b) $C^{-1}(20)$ is the number of minutes you would have to talk to get a \$20 phone bill.

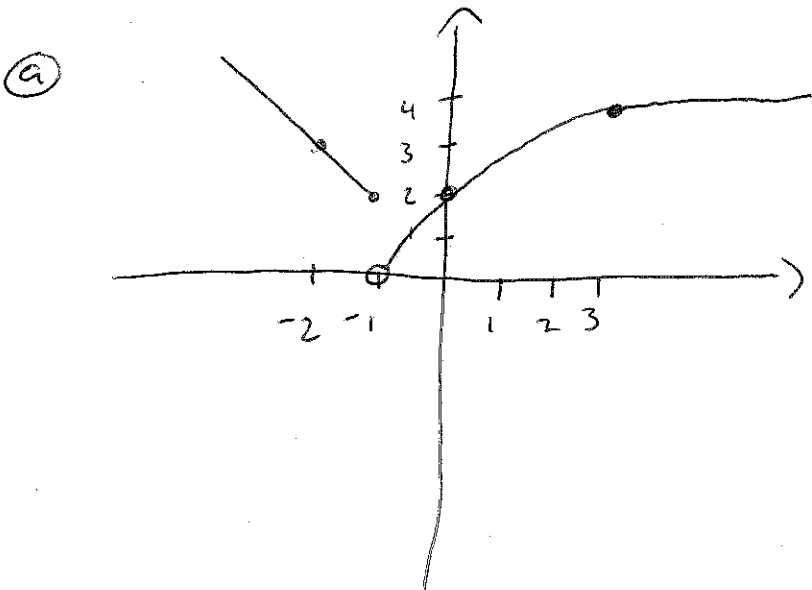
(c) You talk between 0 and 500 minutes in a given month.

Problem 2.

Let $f(x)$ be the function defined as follows:

$$f(x) = \begin{cases} 1-x & x \leq -1 \\ 2\sqrt{x+1} & -1 < x < 3 \\ 4 & x \geq 3 \end{cases}$$

- (a) [5pts.] Sketch a graph of $f(x)$.
(b) [5pts.] On what intervals is f increasing? Decreasing?



(b) Decreasing on $(-\infty, -1)$ (or $(-\infty, -1]$)

Increasing on $(-1, \infty)$ or on $(-1, 3)$

both answers accepted

Problem 3.

Find a formula for each of the following lines.

- (a) [5pts.] The line perpendicular to the line $h(t) = 3t + 4$ and passing through the point $(3, 1)$.
- (b) [5pts.] The line describing your elevation in the following situation: You are climbing a mountain. You climb 40 feet per minute. Five minutes after you start, you are 1600 feet above sea level.

(a) $m = -\frac{1}{3}$

$$y = -\frac{1}{3}x + b$$

$$1 = -\frac{1}{3}(3) + b$$

$$1 = -1 + b$$

$$2 = b$$

$$y = -\frac{1}{3}x + 2$$

(b) $E(t)$ - elevation in feet

t - time in minutes

$$E(t) = 40t + b$$

$$E(5) = 1600$$

$$1600 = 40(5) + b$$

$$1600 = 200 + b$$

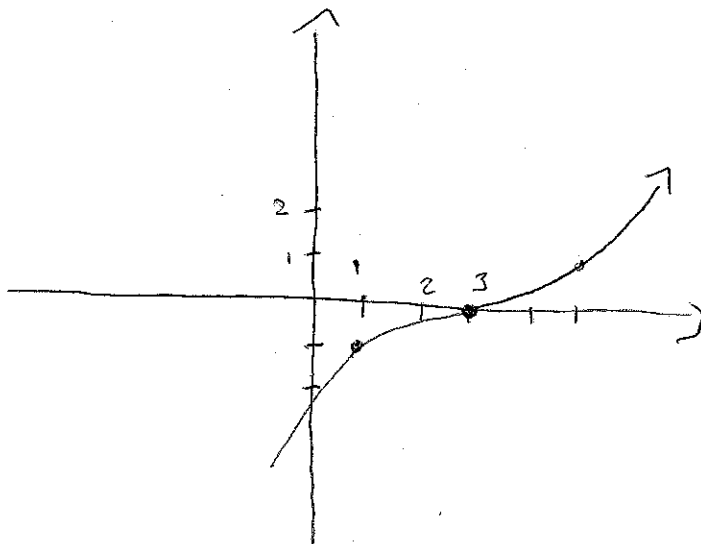
$$1400 = b$$

$$E(t) = 40t + 1400$$

Problem 4.

- (a) [4pts.] Sketch a graph of the function $g(x)$ which is equal to the cubic function $f(x) = x^3$ compressed vertically by a factor of $\frac{1}{8}$ and shifted horizontally to the right by 3.
- (b) [2pts.] Give a formula for the function $g(x)$ you drew in part (a).
- (c) [4pts.] On what intervals is $g(x)$ concave up and concave down?

(a)



(b) $g(x) = \frac{1}{8}(x-3)^3$

(c) Concave up on $(3, \infty)$

Concave down on $(-\infty, 3)$

Problem 5.

Let $f(x) = 3|x + 1| - 4$.

(a) [5pts.] Solve the absolute value equality $f(x) = 8$.

(b) [5pts.] Let $g(x) = \frac{1}{x}$. Find the average rate of change of $g \circ f$ on $[1, 4]$.

① $3|x+1| - 4 = 8$

$$3|x+1| = 12$$

$$|x+1| = 4$$

$$x+1=4 \quad x+1=-4$$

$$\boxed{x=3 \text{ or } x=-5}$$

② $g \circ f(1) = g(3|2|-4) = g(6-4) = g(2) = \frac{1}{2}$

$$g \circ f(4) = g(3|5|-4) = g(15-4) = g(11) = \frac{1}{11}$$

Average Rate of change = $\frac{g \circ f(4) - g \circ f(1)}{4-1} = \frac{\frac{1}{11} - \frac{1}{2}}{3} = \frac{\frac{2-11}{22}}{3} = \frac{-9}{22 \cdot 3}$

$\rightarrow \boxed{\frac{-3}{22}}$